

# Electroweak radiative effects in the single $W$ -production at Tevatron and LHC

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## Abstract

An alternative calculation of the lowest order electroweak radiative corrections (EWC) to the single  $W$ -boson production in hadron-hadron collision in the framework of the quark parton model without any absorption of the collinear quark singularity into the parton distributions is carried out. Numerical analysis under Tevatron and LHC kinematic conditions is performed.

## 1 Introduction

To define the experimental value of the  $W$ -boson mass  $m_W$  with a high precision it is necessary to take into account the radiative effects [1]. Such calculations for the single  $W$ -production at hadron-hadron collider experiments already have been done. So in Ref. [2] radiative effects from the final state photon emissions to the single  $W$ -production were investigated. Basing on more accurate calculation of EWC to the  $W$ -production in a general 4-fermion process [3] the lowest order corrections in hadronic collisions have been calculated in Refs. [4, 5], where the contributions of both initial and final state radiations have been taken into account. The total EWC to the polarization observables of  $W$ -boson production at RHIC within the covariant approach [6] were estimated by one of us (V.Z.) in Refs. [7, 8]. The multiphoton radiation in leptonic  $W$ -boson decays was found in a recent paper [9].

In the present report the new explicit formulae for EWC to the inclusive single  $W$ -production in hadron-hadron collisions are presented. For extraction of infrared singularity the covariant method [6] was used. It is well-known that the modern fits to the quark distributions do not include the EWC. That is why, like Refs. [7, 8], performing the calculation we leave the quark mass singularity without any changes, therefore in spite of Refs. [4, 5] our results are essential depended on the quark masses.

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This report is organized as follows. In Sect. 2 the Born approximation for the single  $W$ -production is presented. In Sect. 3 the total lowest order EWC are considered. The detail numerical results are available in Sect. 4. The conclusions are given in Sec. 5.

## 2 Kinematics and Born approximation

The process of the single  $W$ -boson production in hadron-hadron collision

$$p + p(\bar{p}) \rightarrow W^\pm + X \rightarrow l^\pm + X \quad (1)$$

can be described by two pairs of the quark-antiquark subprocesses according to the charge of  $W$ -boson. For the  $W^-$  production we have the processes

$$q_i(p_1) + \bar{q}_{i'}(p_2) \rightarrow W^- \rightarrow l^-(k_1) + \bar{\nu}_l(k_2), \quad (2.a)$$

$$\bar{q}_i(p_1) + q_{i'}(p_2) \rightarrow W^- \rightarrow l^-(k_1) + \bar{\nu}_l(k_2); \quad (2.b)$$

and for  $W^+$  ones read as

$$q_i(p_1) + \bar{q}_{i'}(p_2) \rightarrow W^+ \rightarrow l^+(k_1) + \nu_l(k_2), \quad (3.a)$$

$$\bar{q}_i(p_1) + q_{i'}(p_2) \rightarrow W^+ \rightarrow l^+(k_1) + \nu_l(k_2). \quad (3.b)$$

Here  $p_1, p_2, k_1, k_2$  are the four-momenta of corresponding particles ( $p_1^2 = m_1^2, p_2^2 = m_2^2, k_1^2 = m_l^2$  and  $k_2^2 = 0$ ).

The standard set of Mandelstam variables on the quark-parton level reads

$$s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2, \quad u = (k_1 - p_2)^2, \quad (4)$$

and for hadrons

$$S = (P_1 + P_2)^2, \quad T = (P_1 - k_1)^2, \quad U = (k_1 - P_2)^2, \quad (5)$$

where  $P_1$  and  $P_2$  are four-momenta of initial hadrons ( $P_1^2 = P_2^2 = m_N^2$ ).

According to the quark parton model (QPM) the substitution  $p_i \rightarrow x_i P_i$  is used. Here  $x_i$  is the fraction of the first ( $i = 1$ ) or second ( $i = 2$ ) hadronic momentum carried by the corresponding struck quark. This procedure will be denoted by operator "hat". As a result in QPM the invariants  $s, t, u$  can be expressed via  $S, T, U$  and  $x_i$  as

$$\hat{s} \approx x_1 x_2 S, \quad \hat{t} \approx x_1 T, \quad \hat{u} \approx x_2 U.$$

At the hadron-hadron collisions the center of parton-parton masses frame has an undetermined motion along the beam direction. Therefore the hadronic Mandelstam variables can be expressed through the standard set of variables in following way:

$$T = -\sqrt{S}|k_{1\perp}|e^{-\eta}, \quad U = -\sqrt{S}|k_{1\perp}|e^{\eta},$$

where  $\sqrt{S}$  is a centre-of-mass energy,  $|k_{1\perp}| \equiv k_{1T}$  is a transversal to the beam direction component of the detected particle four-vector and  $\eta$  is a pseudorapidity.

After integration over azimuth angle ( $d^3k_1/k_{10} \Rightarrow \pi d\eta dk_{1\perp}^2$ ), the hadron-hadron cross section can be presented as the sum over all flavors of quarks and antiquarks

$$\frac{d\sigma_B}{d\eta dk_{1\perp}^2} = \sum_{i,i'} \int dx_1 dx_2 f_i(x_1, Q^2) \Sigma \delta(x_2 + \frac{x_1 T}{x_1 S + U}) = \sum_{i,i'} \int_{x_1^m}^1 dx_1 f_i(x_1, Q^2) \Sigma_0, \quad (6)$$

where

$$\Sigma = \frac{\pi \alpha^2}{4 N_c s_w^4} \frac{|V_{ii'}|^2 \hat{B}_{ii'} \hat{\Pi}_l \hat{\Pi}_l^+ f_{i'}(x_2, Q^2)}{\hat{s}(x_1 S + U)}, \quad \Sigma_0 = \Sigma|_{x_2=x_2^0}, \quad (7)$$

and

$$\Pi_l = \frac{1}{s - m_W^2 + i m_W \Gamma_W} \quad (8)$$

is  $W$ -propagator ( $\Gamma_W$  is the  $W$ -boson width). The lower limit of integration in (6) reads  $x_1^m = -U/(S+T)$ , and the other variables are defined in the following way:  $x_2^0 = -x_1 T/(x_1 S + U)$ ,  $N_c = 3$  is the color factor,  $V_{ii'}$  is CKM matrix element,  $f_i(x_i, Q^2)$  are the spin averaged quark densities,  $Q^2$  is a typical transfer momentum square in the partonic reaction,  $B_{ii'} = u^2$  ( $t^2$ ) for (2.a) and (3.b) ((3.a) and (2.b)) subprocesses.

### 3 The lowest order radiative corrections

The lowest order radiative corrections to the process (1) can be presented as a sum of contributions appearing both from the additional virtual particles (the V-contributions, see Fig. 1) and the real photon emissions (the R-contributions, see Fig. 2):

$$\frac{d\sigma_{RC}}{d\eta dk_{1\perp}^2} = \frac{d\sigma_V}{d\eta dk_{1\perp}^2} + \frac{d\sigma_R}{d\eta dk_{1\perp}^2}. \quad (9)$$

Notice that the V- and R- contributions include infrared (IR) divergences while the sum of them (9) has to be IR free. Therefore one of the main aim of the radiative corrections calculation consists in the cancellation of the IR divergences correctly. Since as it will be presented below the final expressions for the V-contribution factorize in front of the Born cross section and we integrate inside of this factor analytically, the extraction of the IR-part from this contribution can be performed in a simple way. At the same time dealing with radiated process we need to integrate over an unobservable particle (a photon) phase space. Therefore for the extraction of the IR-part from the R-contribution it is necessary to make some manipulation with integrand expression that will be presented below too. Now consider the V- and R-contribution in details.

The V-contribution is proportional to the Born one and could be written as

$$\frac{d\sigma_V}{d\eta dk_{1\perp}^2} = \sum_{i,i'} \int dx_1 f_i(x_1, Q^2) \hat{\delta}_V^{ii'}|_{x_2=x_2^0} \Sigma_0. \quad (10)$$

Here the factor  $\delta_V^{ii'}$  consists of the seven terms

$$\delta_V^{ii'} = \delta_W + \delta_{Vl} + \delta_{Vq} + \delta_{Sl} + \delta_{Sq} + \delta_{\gamma W}^{ii'} + \delta_{ZW}^{ii'}, \quad (11)$$

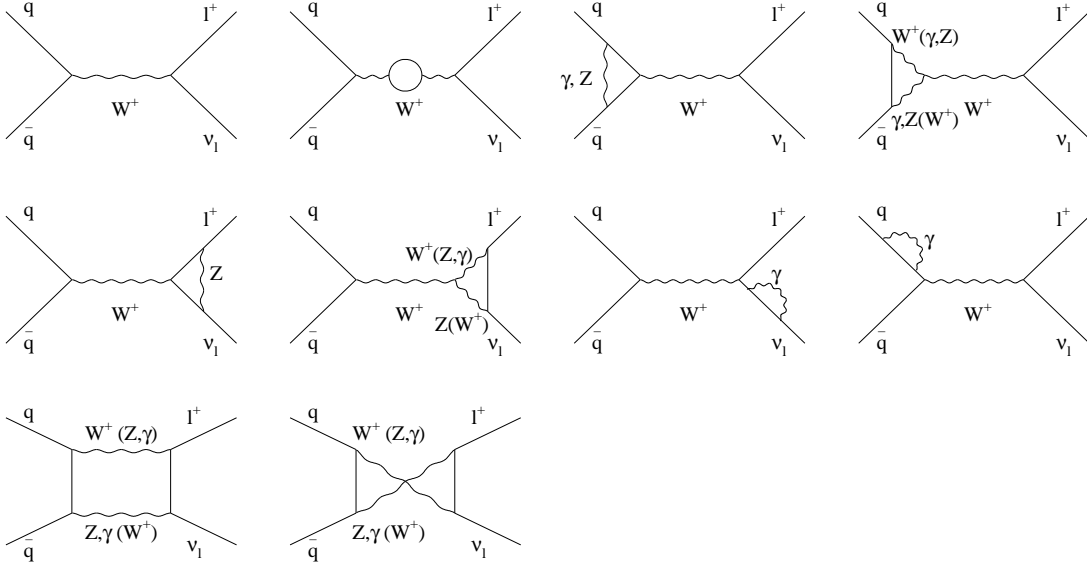


Figure 1: *Born and the virtual one-loop diagrams for  $q\bar{q} \rightarrow l^+\nu$  process. The contribution to the self-energy of  $W$ -boson is symbolized by the empty circle.*

which are contributions of the one-loop diagrams (see Fig.1) to the cross section (1): the  $W$ -boson self-energy  $\delta_W$ ; the leptonic vertex  $\delta_{Vl}$ ; the quark vertex  $\delta_{Vq}$ ; the neutrino self energy  $\delta_{Sl}$ ; the up-quarks self energy  $\delta_{Sq}$ ; the  $\gamma W$ - and  $ZW$  boxes  $\delta_{\gamma W}^{ii'}$   $\delta_{ZW}^{ii'}$  respectively. The explicit expressions for each term can be found in Refs. [10, 11].

The IR-part of the V-contribution can be presented by the formula (10), with the following replacement  $\delta_V^{ii'} \rightarrow \delta_V^{IR}$ , where

$$\begin{aligned} \delta_V^{IR} = & \frac{\alpha}{2\pi} \log \frac{s}{\lambda^2} (Q_l^2 + Q_i^2 + Q_{i'}^2 - Q_i Q_{i'} \log \frac{s^2}{m_i^2 m_{i'}^2} + Q_l Q_{i'} \log \frac{t^2}{m_i^2 m_l^2} \\ & - Q_l Q_{i'} c_l \log \frac{u^2}{m_{i'}^2 m_l^2}), \end{aligned} \quad (12)$$

$Q_j$  is the charge of the fermion  $j$  expressed in the units of the elementary charge  $e = \sqrt{4\pi\alpha}$  (e.g.  $Q_u = +2/3$ ,  $Q_d = -1/3$ ), and  $c_l = +1$  ( $-1$ ) for (2.a) and (3.b) ((3.a) and (2.b)) subprocesses. The rest part of the V-contribution to the cross section contains finite correction

$$\delta_V^{ii'} - \delta_V^{IR} = \delta_V^{ii'}(\lambda^2 \rightarrow s). \quad (13)$$

The contribution of bremsstrahlung

$$p + p(\bar{p}) \rightarrow W^\pm + X \rightarrow l^\pm + \gamma + X \quad (14)$$

to the process (1) after extraction of IR divergence by the method described in Ref. [6] can be presented as a sum of the infrared dependent and infrared free parts

$$\frac{d\sigma_R}{d\eta dk_{1\perp}^2} = \frac{d\sigma_R^{IR}}{d\eta dk_{1\perp}^2} + \frac{d\sigma_R^F}{d\eta dk_{1\perp}^2}. \quad (15)$$

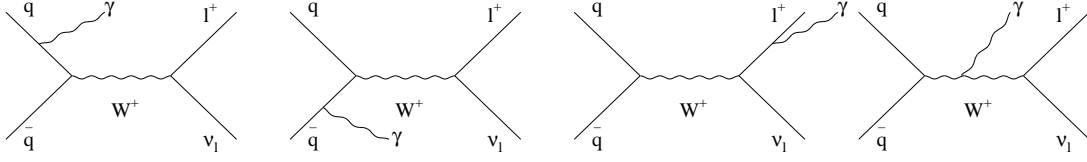


Figure 2: *Bremsstrahlung diagrams for  $q\bar{q} \rightarrow l^+\nu\gamma$  process.*

Summing up IR-parts of the V- and R- contributions we get

$$\frac{d\sigma_R^{IR}}{d\eta dk_{1\perp}^2} + \frac{d\sigma_V^{IR}}{d\eta dk_{1\perp}^2} = \sum_{i,i'} \int dx_1 f_i(x_1, Q^2) \frac{\alpha}{2\pi} \Sigma_0 \hat{J}(t, 0) \log \frac{\tilde{v}^2 \hat{s}_0}{v_{max}^2}, \quad (16)$$

i.e. IR divergence has canceled successfully. Here

$$J(t, v) = Q_l^2 - c_l Q_l Q_i \log \frac{t^2}{m_l^2 m_1^2} + c_l Q_l Q_{i'} \log \frac{u^2}{m_l^2 m_2^2} + Q_i^2 - Q_i Q_{i'} \log \frac{s^2}{m_1^2 m_2^2} + Q_{i'}^2. \\ v_{max} = x_1(S + T) + U, \quad \tilde{v} = -\frac{T x_1}{\sqrt{S}} \left(1 + \frac{TU}{(x_1 S + U)^2}\right). \quad (17)$$

After IR-terms extraction the finite part of the R-contribution (so called "hard" photon contribution) reads

$$\frac{d\sigma_R^F}{d\eta dk_{1\perp}^2} = \sum_{i,i'} \int dx_1 dx_2 f_i(x_1, Q^2) f_{i'}(x_2, Q^2) \hat{\Sigma}_R^F. \quad (18)$$

The kinematic regions for integration over  $x_1$  and  $x_2$  are restricted by the following region

$$-\frac{U + m_N^2 - m_l^2}{S + T + m_N^2} \leq x_1 \leq 1, \quad x_2^0 \leq x_2 \leq 1,$$

and  $\Sigma_R^F$  reads

$$\Sigma_R^F = \frac{\alpha^3}{8N_c s_w^4 s} |V_{ii'}|^2 (Q_l^2 \Pi_l \Pi_l^+ V_l + Q_l \Re[\Pi_l] V_{lq} + V_q + Q_l \Pi_l \Pi_l^+ V_{lw} + \Re[\Pi_l] V_{qw} + \Pi_l \Pi_l^+ V_w). \quad (19)$$

The indexes of the  $V$ -terms correspond to radiated leg ( $l$  — final lepton,  $q$  — initial quarks,  $w$  —  $W$ -boson), as well as the double index corresponds to the same interference term.

## 4 Numerical results

The presented above formulae allows us to estimate numerically the lowest order radiative effects for the  $W$ -production at the Tevatron ( $p\bar{p}$  collisions,  $\sqrt{S}=1.8\text{TeV}$ ) and LHC ( $pp$  collisions,  $\sqrt{S}=14\text{TeV}$ ) kinematic conditions.

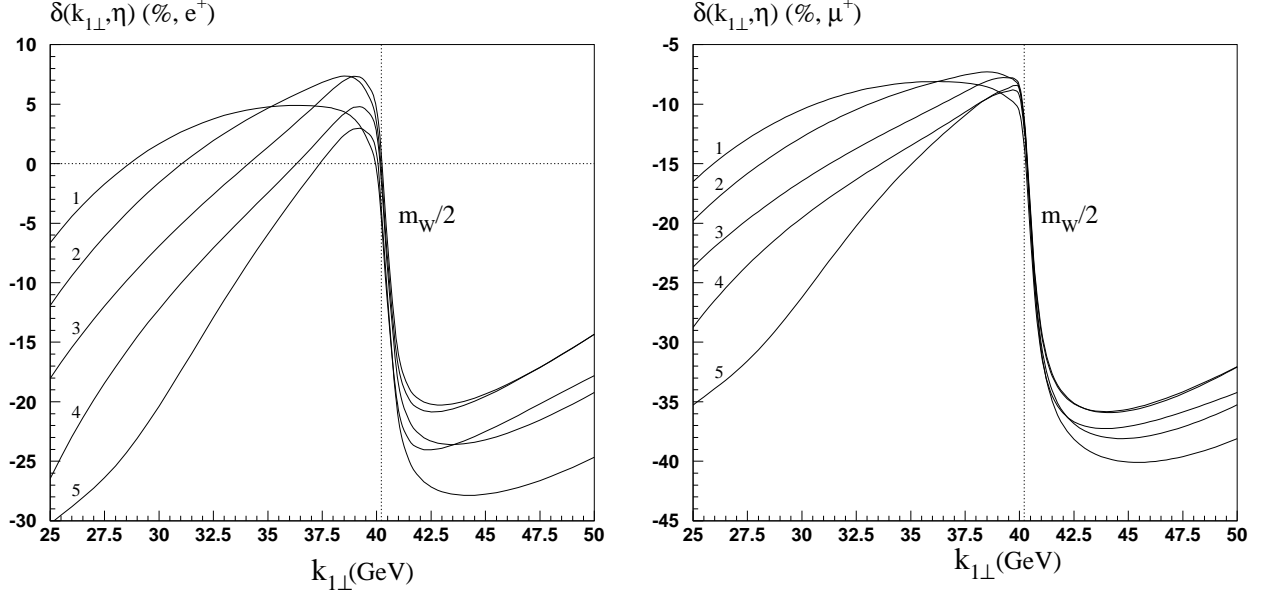


Figure 3: *Relative corrections to the lepton transverse momentum distribution at  $\sqrt{S} = 1.8$  TeV, (Tevatron) for different pseudorapidity  $\eta$ . The numbers 1, 2, 3, 4, 5 correspond to the lines with  $\eta = -2, -1, 0, 1, 2$  respectively. The quark masses are defined by (21).*

The performed analysis has shown that the differences between the relative corrections which are defined as a ratio of EWC to the Born contribution

$$\delta(k_{1\perp}, \eta) = \frac{d\sigma_{RC}/d\eta dk_{1\perp}}{d\sigma_B/d\eta dk_{1\perp}} \quad (20)$$

calculated within MRS98 [12], GRV98 [13], CTEQ [14] parton distributions are not essential and MRS98 will be used for the numerical analysis in this section (we use as well as in Ref.[4]  $Q^2 = m_W^2$ ).

For the numerical analysis we use the rule of the naive QPM  $p_q = x_q P_N$ , when the transversal component of the quark momentum inside of the hadron can be dropped that leads to the following relation

$$m_q = x_q m_N, \quad (21)$$

where  $x_q$  is an argument of corresponding parton distribution. It should be noted that numerical estimations of EWC with this choice for quark masses and with  $m_u = 5$  MeV,  $m_d = 9$  MeV give very close results.

To estimate dependence of radiative effects on the leptonic masses numerically, our plots were constructed for  $e^+X$  and  $\mu^+X$  final states. From the Figs. 3 and 4 one can see that corrections have a positive sign only in the small region of the positron transverse momentum at Tevatron kinematics and negative sign for the whole regions of the antimuon transverse momentum at Tevatron and LHC ones. This situation can be explained as suppression of the hard photon emission (whose contribution is always positive) by the production of the additional virtual particles (whose contributions are negative). The difference in the behavior of EWC for  $e^+X$  and  $\mu^+X$  final states appears as the contributions of collinear singularities from the final lepton radiation which are proportional to  $\log(s/m_l^2)$ . The other interesting

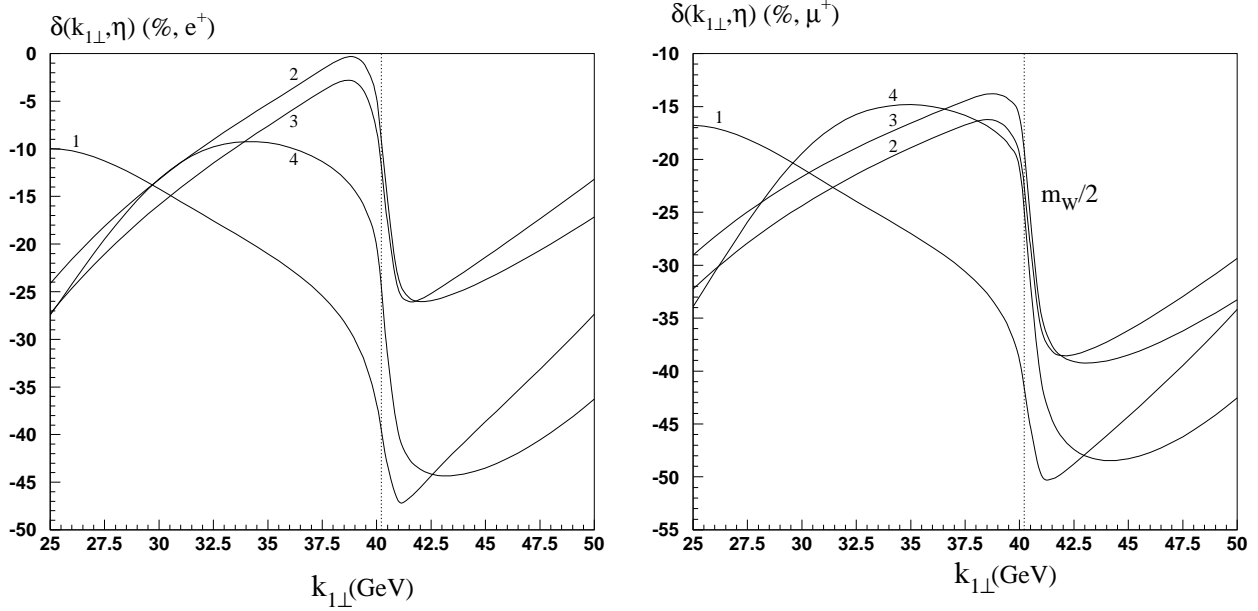


Figure 4: *Relative corrections to the lepton transverse momentum distribution at  $\sqrt{S} = 14$  TeV, (LHC) for different pseudorapidity  $\eta$ . The numbers 1, 2, 3, 4, 5, 6 correspond to the lines with  $\eta = -3, -1.3, 0, 1.3, 3, 5$  respectively. The quark masses are defined by (21).*

feature consists in rather high sensibility of the transverse momentum distribution to the changes of pseudorapidity. The steep fall of EWC near  $k_{1T} = m_W/2$  comes as the reflection of the resonance behavior.

It is shown at Fig. 5 that after integration over the pseudorapidity ( $-1.2 < \eta < 1.2$  for Tevatron and  $-5 < \eta < 5$  for LHC) a positive sign of corrections

$$\delta(k_{1\perp}) = \frac{d\sigma_{RC}/dk_{1\perp}}{d\sigma_B/dk_{1\perp}} \quad (22)$$

survives only for the positron final state at Tevatron kinematics and these corrections reach  $\sim 6\%$  at  $k_{T1} = 39$  GeV.

According to Fig. 6 EWC decrease the cross section at the peak  $k_{1T} = m_W/2$  of the transverse lepton momentum spectrum by about 11% for  $\mu^+$  final state, increase by about 4% for  $e^+$  final states for Tevatron, and decrease the cross section by about 12% and 24% in  $e^+$  and  $\mu^+$  final states respectively for LHC kinematics.

## 5 Conclusions

Using the covariant approach [6] and the formulae for additional virtual particle productions [11] the explicit expressions for the lowest order EWC to the single  $W$ -production in hadron-hadron collisions are obtained.

The main features of our calculation are:

- the final expressions for the lowest order EWC do not depend on any unphysical cutoff parameters;

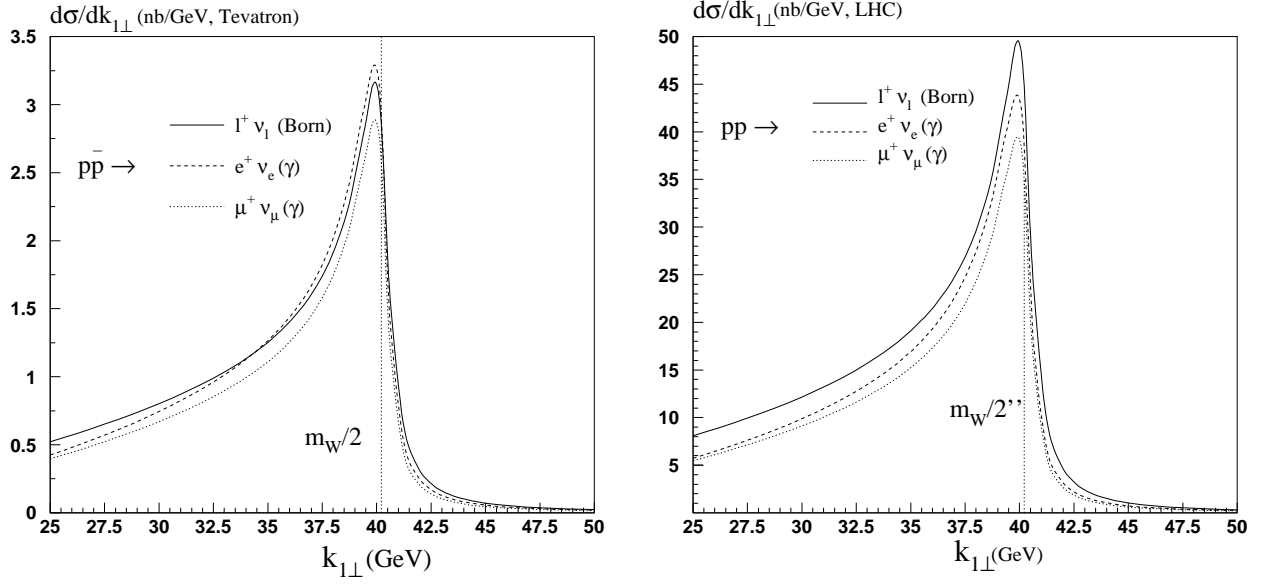


Figure 5: *The lepton transverse momentum distribution on the Born level and with taking into account EWC at  $\sqrt{S} = 1.8$  TeV,  $|\eta| < 1.2$  (Tevatron) and  $\sqrt{S} = 14$  TeV,  $|\eta| < 5$  (LHC). The quark masses are defined by (21).*

- during the calculation the quark mass singularity is kept without any changes;
- The numerical analysis performed for Tevatron and LHC kinematics has shown that
- EWC are very sensitive to the pseudorapidity changes;
  - the hard photon emission is essential suppressed by the additional virtual particle production. As a result in general EWC to the single  $W$ -production has a negative sign almost in the whole region of  $W$ -experiments at Tevatron and LHC machines.

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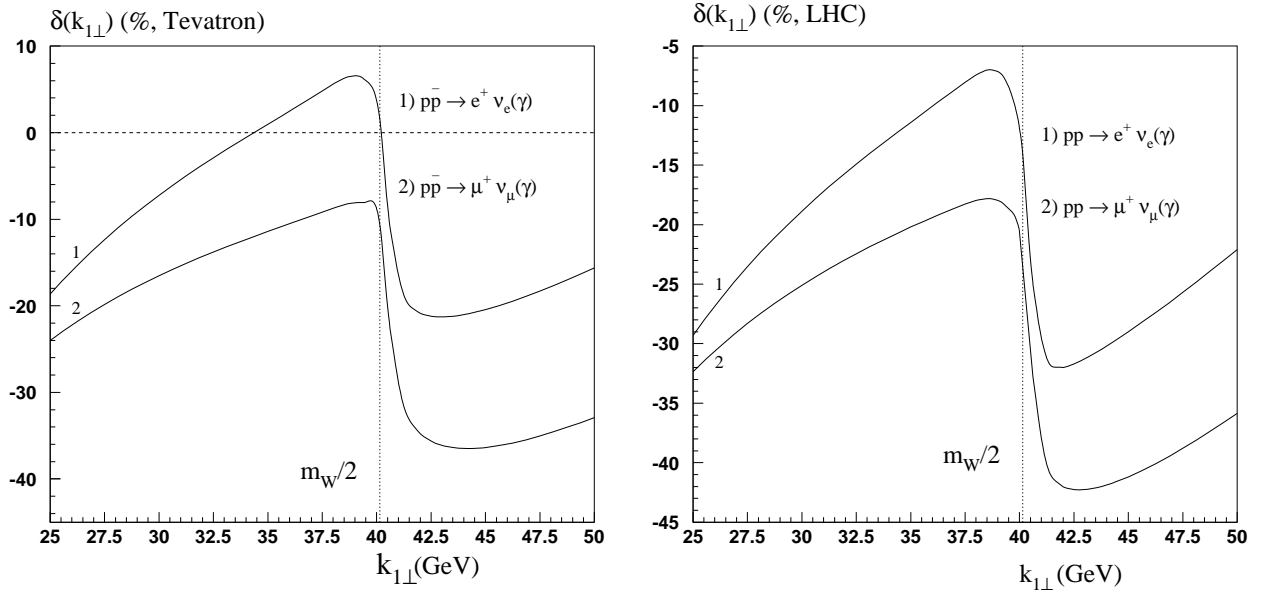


Figure 6: *Relative corrections to the lepton transverse momentum distribution at  $\sqrt{S} = 1.8$  TeV,  $|\eta| < 1.2$  (Tevatron) and  $\sqrt{S} = 14$  TeV,  $|\eta| < 5$  (LHC). The quark masses are defined by (21).*

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